Definition
What is a Composite Element?

- When the materials become complementary:
  - A composite element is a structural member composed of two or more dissimilar materials joined together to act as a unit.
  - There are times when a specific material may provide good incentives to be used for one reason, but it's qualities may be inadequate,
  - or perhaps there may be a scenario when an element may have geometric restrictions that will not allow it to perform the necessary task.
  - Geometry & material quality can work together to provide the necessary result!

How do these materials work in tandem?
WHAT IS THE SECRET FORMULA

▪ Ideal Scenario:
  ▪ The main goal is to maximize the benefit of each of the materials.
  ▪ A material that takes high stress would better be placed there where the stress is maximum, while a different material will be filling gaps!
  ▪ Take the case of a beam:
    ▪ Use wood as web
    ▪ Use steel as flange

WHAT IS THE SECRET FORMULA

▪ How does it work?
  ▪ The usual formulae that were applied for stress are still applicable, but there is a significant change.
  ▪ The designer should translate one material to an equivalent geometry for another!
  ▪ This is done by using an appropriate ratio of Young's moduli of Elasticity.
    ▪ e.g. steel has an E value of 29,000$^{\text{psi}}$ whilst a certain type of wood may have E at 1000$^{\text{psi}}$.
  ▪ So that should be compensated by equivalent virtual transformation of the cross sectional area of one material into another.
Example 1:

- A simply supported wooden beam (at negligible dead load) has a steel plate attached on the bottom. The two fully develop as a composite element.
- Determine the tensile stress in the steel
Composite Beam Design (Wood & Steel)

Problem Statement:
A simply supported wooden beam (at negligible dead load) has a steel plate attached on the bottom. The two fully develop as a composite element. Determine the tensile stress in the steel.

Unbraced Length:
\[ L_b := 12 \text{ ft} \]
Length on left of point load
\[ L_1 := 5 \text{ ft} \]
Design Point Load
\[ P_u := 8 \text{ kip} \]
Base width
\[ b_w := 6 \text{ in} \]
Steel Young's Modulus of Elasticity
\[ E_s := 29000 \text{ ksi} \]
Wood Young's Modulus of Elasticity
\[ E_w := 1600 \text{ ksi} \]
Height of wood beam
\[ h := 8 \text{ in} \]
Steel plate thickness
\[ t_{pl} := \frac{3}{16} \text{ in} \]

Solution:

1) Determining the reactions
\[ R_B := \frac{P_u L_1}{L_b} \]
\[ R_B = 3.33 \text{ kip} \]
\[ R_A := P_u - R_B \]
\[ R_A = 4.67 \text{ kip} \]
\[ M_u := R_A L_1 \]
\[ M_u = 23.33 \text{ k}' \]

2) Calculating the Moment

3) Calculating the Moment of Inertia
a) It is necessary to transform the wood to what form it would have if it was steel and produced the same resistance to the loads as it does by being wood. This is based on the ratio of Steel and Wood Young's Modulus of Elasticity.
\[ n := \frac{E_s}{E_w} \]
\[ n = 18.13 \]

b) So the transformed width of the wood would be equivalent to \( b_w \) multiplied by \( n \):
\[ b_{w\text{,transf}} := \frac{b_w}{n} \]
\[ b_{w\text{,transf}} = 0.331 \text{ in} \]

c) Then it is necessary to determine the location of the neutral axis in order to calculate the Moment of inertia of the composite element. Taking measurements from the bottom of the beam:
\[ A_w := b_{w\text{,transf}} t_{pl} \]
\[ A_w = 1.12 \text{ in}^2 \]
\[ A_s := b_w t_{pl} \]
\[ A_s = 1.12 \text{ in}^2 \]
\[ n_A := \frac{A_s}{A_s + A_w} \]
\[ n_A = 0.38 \]
\[ A_{w\text{,transf}} := A_w \left( n_A + \frac{t_{pl}}{2} \right) \]
\[ A_{w\text{,transf}} = 2.65 \text{ in}^2 \]
\[ A_{w\text{,transf}} = b_{w\text{,transf}} t_{pl} \]
\[ A_{w\text{,transf}} = 2.65 \text{ in}^2 \]
\[ NA := \frac{A_s t_{pl}}{2} + A_{w\text{,transf}} \left( t_{pl} + \frac{h}{2} \right) \]
\[ NA = 2.97 \text{ in} \]

\[ I_{st} := \frac{b_{w\text{,transf}} t_{pl}^3}{12} + A_s \left( NA - \frac{t_{pl}}{2} \right)^2 \]
\[ I_{st} = 9.29 \text{ in}^4 \]
\[ I_w := \frac{b_{w\text{,transf}} t_{pl}^3}{12} + A_{w\text{,transf}} \left( t_{pl} + \frac{h}{2} - NA \right)^2 \]
\[ I_w = 18.07 \text{ in}^4 \]
\[ I_{tot} := I_{st} + I_w \]
\[ I_{tot} = 27.36 \text{ in}^4 \]
4) Calculating the Stress:

\[ f_b = \frac{M \cdot c}{I} \]

\[ f_b := \frac{M_u \cdot c}{I_{tot}} \]

\[ f_b = 30.36 \text{ ksi} \]

\[ c := \text{NA} \]

NOTE: In case the stress in the wood is required, the value given by the formula will need to be divided by the ratio \( n \) that was determined above so that it would correspond to the area of wood as it is, not the transformed area.
But what happens in Reality?

- Full Development is an issue:
  - There is a problem on how the two would ideally be fully developed so that the stresses can transfer between the two materials.
  - Just imagine how good that glue has to be to not allow slipage and shear between the steel plate and the wood.
  - Holes can be applied to the plate to nail through to the wood, …
    - … but in the industry they find it easier to create a “flitch beam”

Advantages/Disadvantages

- The advantage is in the ease of manufacturing

- The disadvantage is that there is plenty of steel in the N/A area, and not that much at the area of high flexural stress.

- Essentially, instead of having steel as flange, we have steel as web!

- But does it help?
Example 2:

Using the previous example, process the same geometry for wood with a same thickness steel plate in the middle generating a flitch beam.