MOMENTS

A lecture assembled for the course on
Statics and Strength of Materials

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Definition of Moment

Relation of Force to Distance

- Moments:
  - We have encountered the condition where the forces originate at one specific point. What happens if an object is subjected to forces that do not meet at one point?
  - In order for an object to be in a static condition the sum of the forces in all component directions (x, y, and z) need to be equal to zero, but if they are not meeting at one point the result will be rotation instead of translation. The possibility of translation is also extant, but there shall be rotation with it.
  - Think of the action that takes place when a driver shifts gears. There is a force applied toward a certain direction by the driver's arm, but, the shift stick being hinged on the bottom is subjected to an analogous reaction. The two forces do not meet at a point, and thus there is rotation.

$$M = F \cdot D$$

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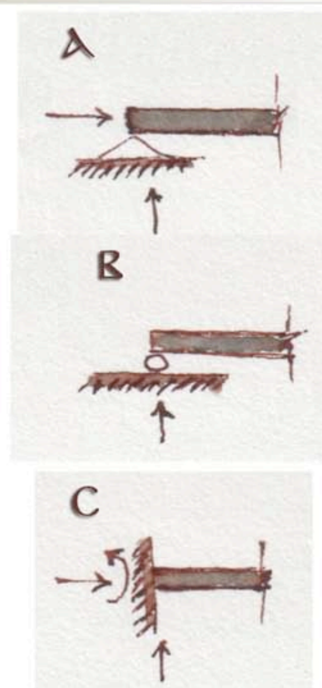
Relation of Force to Distance

- Moments:
  - However, there may be conditions that would allow such moment to be generated, yet do not allow rotation of the element because of the type of connections to which the element is subjected.
  - In the previous scenario, the gear shift is only connected at the bottom and the connection allows rotational action to take place. That is a so called hinged connection which belongs to the category of flexible connections.
  - Commonly we consider two major types of connections, the rigid and the flexible connections. In the diagram the A and B examples constitute flexible connections. Example A resists horizontal and vertical forces, and Example B supports only vertical reactions. In contrast, Example C which is considered to be a rigid connection. It supports reactions for vertical and horizontal reactions as well as moments.



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Relation of Force to Distance

- Moments:
  - Let's take a look at the application on what could be a quotidian type of activity, such as opening the cabinet doors with springs, or shearing a nail with a pair of snippers.
  - The user in this case, applies force on the levers of the snippers. On the other end, the nail, resisting the shear force, balances the system. The forces applied by the user are not equal to the resistance by the bolt. Do not confuse that with the sum of forces is equal to zero. Yes, the sum is equal to zero if we consider the left lever and the right lever that the user presses. Similarly, the right and left blade of the snippers receive equal resistance.
  - However, the user has a great advantage. The levers have much longer arm than the blades. The moment on each side should be equal to be static. Once the moment on one side is greater, the system is no longer static.

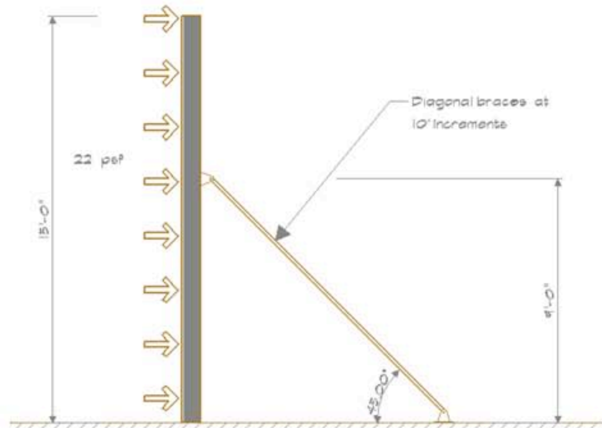
$$M = F \cdot D$$

Example: Wall Supported by Braces

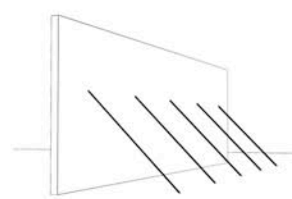
- Problem Statement:
  - A wall is subjected to wind loads and is supported by braces as indicated in the diagram. Determine the axial load that the braces are subjected to:
  - First, we need to determine a good location as a reference point to measure the moments so that we can use the formulae:

$$\Sigma M = 0, \Sigma F_H = 0, \text{ and } \Sigma F_V = 0$$

  - By observation, it is evident that there is no Vertical forces to worry about , and evidently, point "A" is an ideal location to take moments!



A



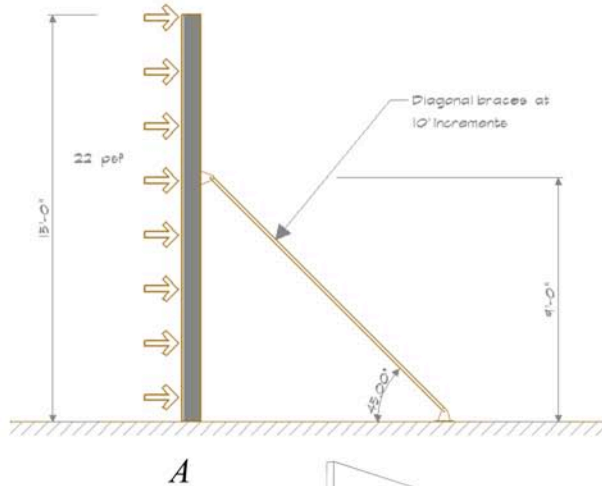
Example: Wall Supported by Braces

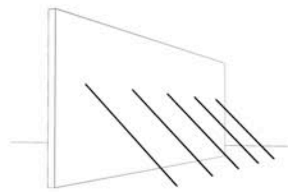
- Solution:
  - We can assume that each bracing assumes a tributary area half way to the next bracing. Therefore, there will be an area of 10' by 15' subjected to 22^{psf} for each bracing:

$$Wind\ load = 150^{ft^2} \cdot 22^{psf} = 3300^{lbf}\ \text{OR}\ 3.3^{K}$$

  - It is also rational to assume that the resultant force will be acting on the center of gravity of the tributary area, thus at a height of 7.5' and horizontally on the same plane where the bracing is located.
  - So if we take the clockwise moment from point "A":

$$M_{A_{cw}} = 3.3^{K} \cdot 7.5^{ft} = 24.75^{K'}$$





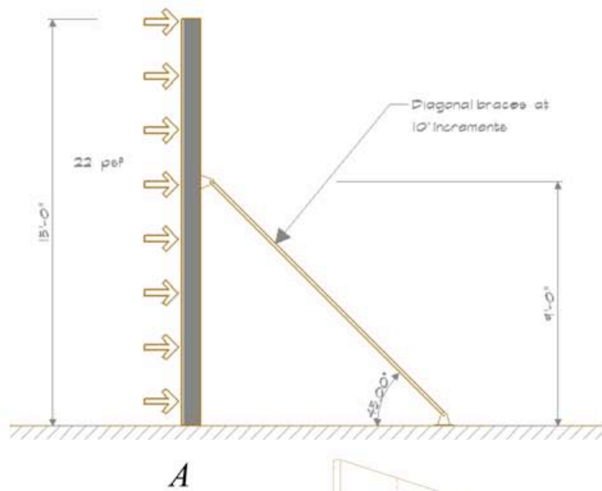
Example: Wall Supported by Braces

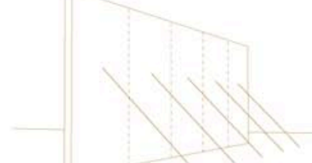
- Solution:
  - The sum of the moments should be equal to zero. Therefore, those 24.75 K' should be countered by a counter clockwise moment of equal magnitude.
  - The braces are set at a height of 9 ft. Therefore the horizontal component of the force that is applied by the braces should equal:

$$F_{Brace_H} = \frac{24.75^{K'}}{9^{ft}} = 2.75^{K}$$

  - By use of trigonometry we can determine that the load assumed by the brace is:

$$F_{Brace} = \frac{2.75^{K}}{\cos(45°)} = 3.89^{K}$$



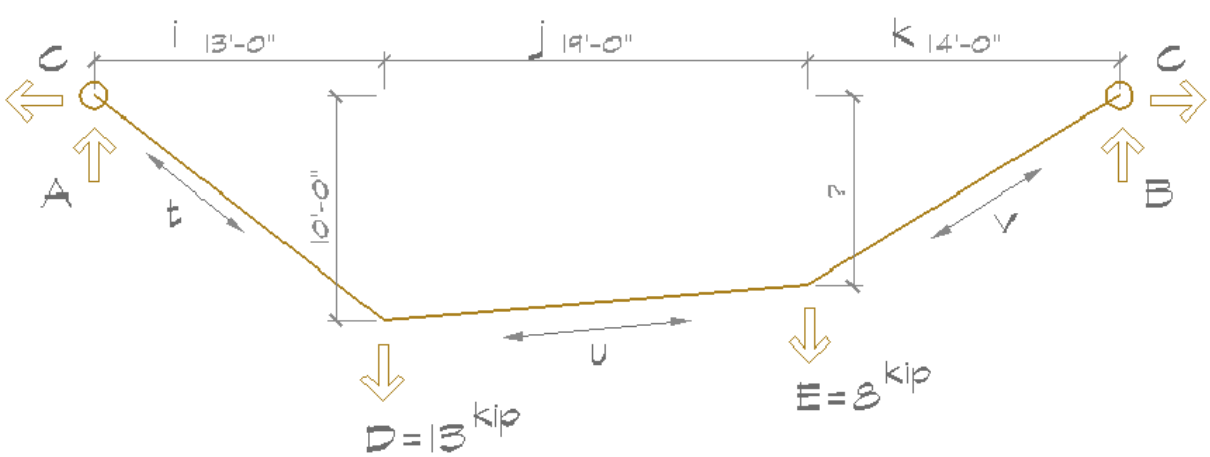


Examining the case of a cable

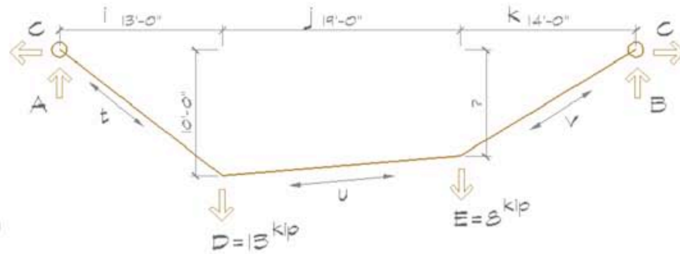
Suppose we have a cable loaded

Cable with suspended loads at various locations:

- *A cable is presented with the given loads at the given locations. Solve for all the unknowns*



SOLUTION FOR CABLE PROBLEM



Given Data:

$i := 13ft \quad j := 19ft \quad k := 14ft \quad x := 10ft \qquad D := 13kip \quad E := 8kip$

Solving for main reactions at cables ends:

Taking moments around point A to solve for reaction "B"

$$B := \frac{[i \cdot D + (i + j) \cdot E]}{(i + j + k)} \qquad B = 9.239 \cdot kip$$

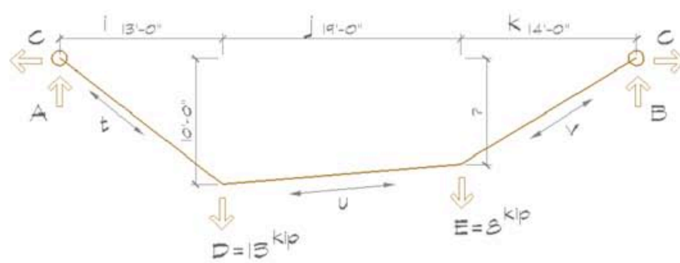
Using the relation that Fy=0 we can solve for reaction "A"

$$A := (D + E) - B \qquad A = 11.761 \cdot kip$$

Horizontal force "C" should be equal on both sides (otherwise we would be moving!). Solving for "C" on left side by taking moments around point "D"

$$C := \frac{i \cdot A}{x} \qquad C = 15.289 \cdot kip$$

SOLUTION FOR CABLE PROBLEM



Using Pythagorean theorem we can solve for tension of cable "t"

$$t := \sqrt{C^2 + A^2} \qquad t = 19.289 \cdot kip$$

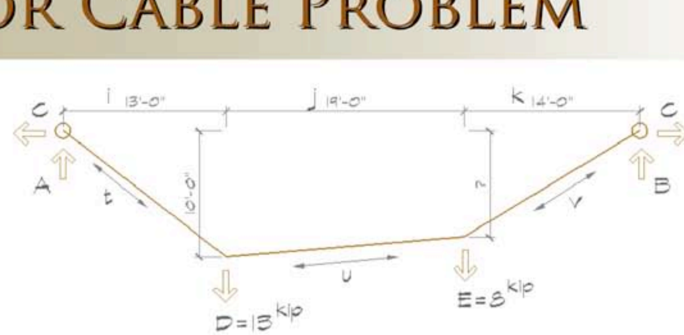
Using Pythagorean theorem we can solve for tension of cable "v"

$$v := \sqrt{C^2 + B^2} \qquad v = 17.864 \cdot kip$$

Taking moments around point "E" we can solve for the depth "y"at that point

$$y \cdot C = k \cdot B \qquad y := k \cdot \frac{B}{C} \qquad y = 8.46 ft$$

Solution for Cable Problem



Knowing that each node should have a sum of forces equal to zero, we can solve for the tension "u" in the cable by solving for either node "D" or node "E". Let's take node "D" and solve the angles, and then based on Fx=0 solve for tension "u" in the cable:

$\text{length}_t := \sqrt{i^2 + x^2}$ $\quad \text{length}_t = 16.401\,\text{ft}$ $\qquad \text{Angle}_{\theta 1} := \text{acos}\left(\dfrac{i}{\text{length}_t}\right)$ $\quad \text{Angle}_{\theta 1} = 37.569\cdot\text{deg}$

$\text{length}_u := \sqrt{j^2 + (x - y)^2}$ $\quad \text{length}_u = 19.062\,\text{ft}$ $\qquad \text{Angle}_{\theta 2} := \text{acos}\left(\dfrac{j}{\text{length}_u}\right)$ $\quad \text{Angle}_{\theta 2} = 4.633\cdot\text{deg}$

$t\cdot\cos(\text{Angle}_{\theta 1}) = u\cdot\cos(\text{Angle}_{\theta 2})$ $\qquad u := \dfrac{(t\cdot\cos(\text{Angle}_{\theta 1}))}{(\cos(\text{Angle}_{\theta 2}))}$ $\qquad u = 15.339\cdot\text{kip}$

Some Reminders

- When working with angles...
  - You may run into situations where it may seem difficult to recognize the geometric configurations in a triangle. All you need is a combination of three data. The only problem will be if all three are angles in which case you cannot determine the size of any side.
  - There are two very important rules to keep in mind. They may come in handy.
  - The Sine rule:

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(B)}$$

  - The Cosine rule

$$a^2 = b^2 + c^2 - 2\cdot b\cdot c\cdot \cos(A)$$