Eccentrically loaded Welded and Bolted Connections

MORGAN STATE UNIVERSITY
SCHOOL OF ARCHITECTURE AND PLANNING

LECTURE XIV

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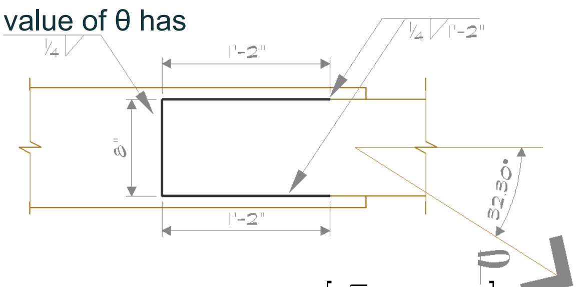
Eccentric Loading

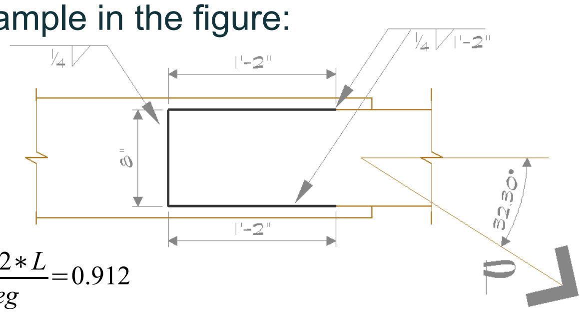
- Let's start with Simple Off Axis Tension
  - The simplified formula we used will not work in this case because the value of θ has changed!



$$\varphi R_n = \varphi * F_{nw} * A_{we} = 0.75\left[0.60 * F_{EXX} * \left(1 + 0.5 * \sin^{1.5}\theta\right)\right] * \left[\frac{\sqrt{2}}{2}\beta * Leg * L\right]$$

Eccentric Loading

- Calculating the capacity of the example in the figure:



$$\beta = 1.2 - \frac{0.002 * L}{Leg} = 0.912$$

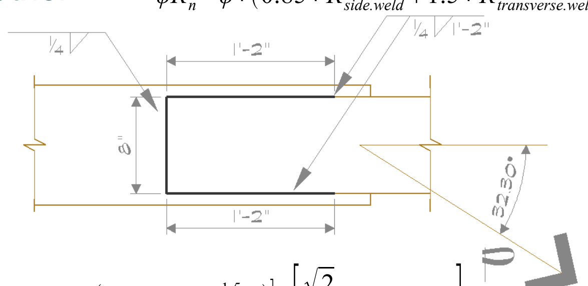
$$\varphi R_n = \varphi * F_{nw} * A_{we} = 0.75\left[0.60 * F_{EXX} * \left(1 + 0.5 * \sin^{1.5}\theta\right)\right] * \left[\frac{\sqrt{2}}{2}\beta * L * Leg\right]$$

$$\varphi R_n = 0.75 * 0.6 * 70^{ksi} * \left(1 + 0.5 * \sin^{1.5} 32.30^o\right) * 0.707 * 0.912 * 36^{inch} * 0.25^{inch} = 218.53^{kip}$$

Eccentric Loading

- BUT, there is another equation...J2-9b

$$\varphi R_n = \varphi * \left(0.85 * R_{side.weld} + 1.5 * R_{transverse.weld}\right)$$



$$\beta = 1.2 - \frac{0.002 * L}{Leg} = 0.912$$

$$\varphi R_n = \varphi * F_{nw} * A_{we} = 0.75\left[0.60 * F_{EXX} * \left(1 + 0.5 * \sin^{1.5}\theta\right)\right] * \left[\frac{\sqrt{2}}{2}\beta * L * Leg\right]$$

$$0.75 * 0.6 * 70^{ksi} * \left(1 + 0.5 * \sin^{1.5} 32.30^o\right) * 0.707 * 0.912 * \left[0.85 * 28^{inch} + 1.5 * 8^{inch}\right] * 0.25^{inch} = 217.31^{kip}$$

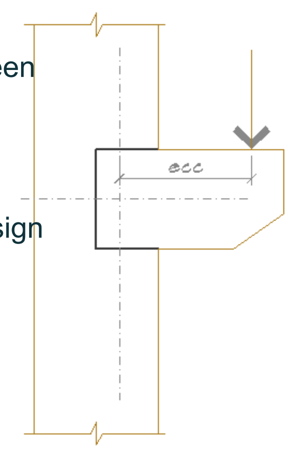
- In which case, actually the largest governs, i.e. 218.53 ^{kips}

Eccentric Loading

- Ec Centric – Ἐκ Κεντρικόν – Off Centered
  - A load that is applied at a distance from the center of the element
  - In case axial loading is anticipated...
- What is the effect?
  - The effect is that of Force multiplied by Distance
  - That results in a moment
  - The effect is simple, but the way it is treated may be a bit more complicated
    - Elements that are set to receive axial loading may end up being subjected to flexural stresses. How does this effect the connections of these elements?

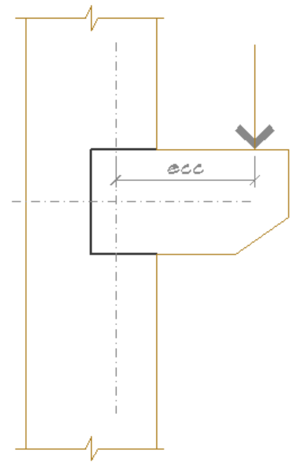
Effects of Eccentric Loading on Welded Connections

- Shear
  - Occurs due to translational movement between surfaces
- Torsion
  - Occurs due to the establishment of a pivotal point (the center of gravity of the specific design in the case of the illustration) and a load that passes at a distance
  - The distance is the eccentricity
- Shear can take place in both linear and curvilinear manner.
- Torsion will cause shear in a curvilinear manner



Elastic Method

- Analysis
  - The elastic method is rather conservative as friction, or slip resistance, between the parts is neglected.
  - The parts are considered perfectly rigid, hence deformation takes place exclusively in the weld
  - The welds are assumed to be perfectly elastic



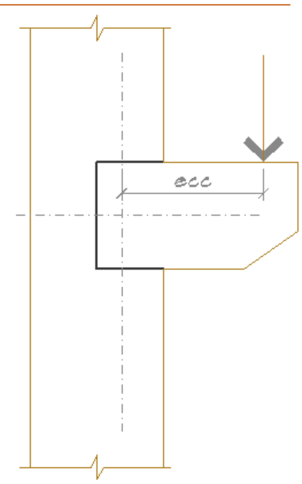
Elastic Method

- The horizontal and the vertical components of the force exerted due to the torsion can be calculated by the following two expressions:

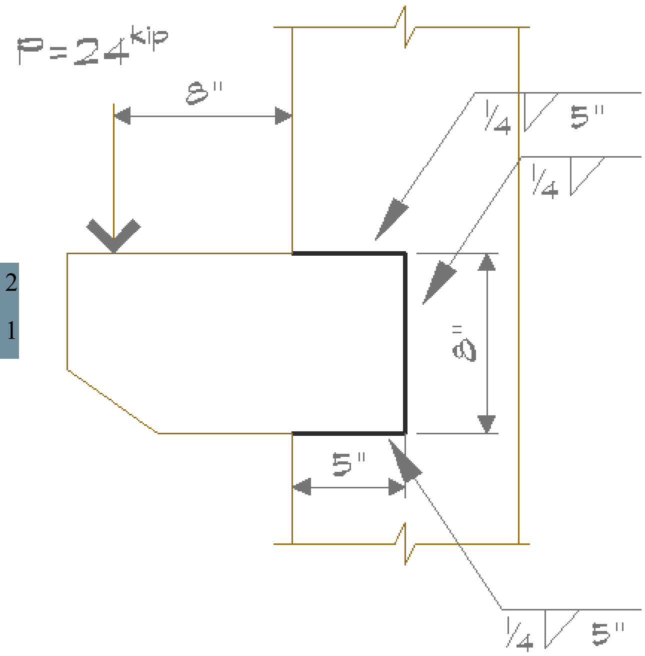
$$r_{muh} = \frac{T * v}{I_p} \qquad r_{muv} = \frac{T * h}{I_p}$$ Eqs. 8-9a, 8-10a

  - Where r_{mu} is the resultant shear due to moment, T is torsion (can consider as the product of P_u and eccentricity), v & h are the vertical and horizontal distances, and I_p is the polar Moment of Inertia

    (slightly modified characters from AISC manual)
  - *For the overall shear effect we use* $r_{mus} = \frac{P_u}{L_{tot}}$



Eccentrically Loaded Welded Bracket Design



Problem Statement:

Determine if the fillet weld for the bracket in the figure is adequate. If not determine the proper size:

Length on x axis: $L_x := 5in$

Length on y axis: $L_y := 8in$

For calculations, leg shall default to 1 inch $Leg := 1in$

Yield Stress: $F_{EXX} := 70ksi$

Applied Design Load $P_u := 24kip$

Legs on x axis: $Legs_x := 2$

Legs on y axis: $Legs_y := 1$

Distance off edge: $d := 8in$

Length total: $L_{tot} := Legs_x \cdot L_x + Legs_y \cdot L_y$ $\quad L_{tot} = 18 \cdot in$

Solution:

1) Determining the area of the weld (total and along the x and Y axes) and the location of the center of gravity along the x axis. (Along Y axis it is the midpoint due to symmetry)

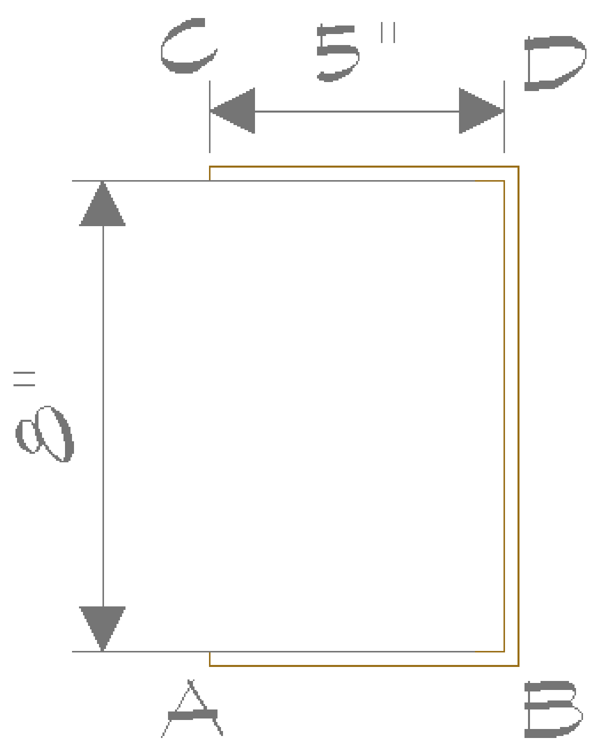
$A_x := Legs_x \cdot L_x \cdot Leg$ $\quad A_x = 10 \cdot in^2$ $\qquad A_y := Legs_y \cdot L_y \cdot Leg$ $\quad A_y = 8 \cdot in^2$ $\qquad A_{tot} := A_x + A_y$ $\quad A_{tot} = 18 \cdot in^2$

$$x_{bar} := \frac{A_x \cdot \frac{L_x}{2}}{A_{tot}} \qquad x_{bar} = 1.389 \cdot in \qquad \frac{8Leg^3}{12} = 0.667\ in^3$$

$$I_x := \frac{Leg\,L_y^3}{12} + \frac{Legs_x \cdot Leg\,Leg^3}{12} + Legs_x \cdot Leg \cdot L_x \cdot \left(\frac{L_y}{2}\right)^2 \qquad I_x = 202.83 \cdot in^4$$

$$I_y := \frac{L_y\,Leg^3}{12} + \left(L_y \cdot Leg\right) \cdot x_{bar}^2 + \frac{Legs_x \cdot Leg\,L_x^3}{12} + Legs_x \cdot \left(Leg \cdot L_x\right) \cdot \left(\frac{L_x}{2} - x_{bar}\right)^2 \qquad I_y = 49.28 \cdot in^4$$

Note: The units of "I" are inches^4 but this method applies inches^3. Therefore, in the process of calculating the polar moment of inertia, which is the addition of the Ix and the Iy, the sum is divided by the value of 1 inch to achieve the unit value of inches^3



$$I_p := \frac{I_x + I_y}{1in} \qquad I_p = 252.11 \cdot in^3$$

2) Calculating the capacity of the weld <u>per inch</u>

$$\beta := \min\left(1, 1.2 - 0.002 \cdot \frac{L_{tot}}{Leg}\right) \qquad \beta = 1$$

$$\Phi R_n := .3182 \cdot Leg \cdot F_{EXX} \cdot \beta \qquad \Phi R_n = 22.274 \cdot \frac{kip}{in}$$

3) Determining the forces on ends B and D

$$Ecc := d + L_x - x_{bar} \qquad Ecc = 11.611 \cdot in$$

$$r_{muh} := \frac{P_u \cdot Ecc \cdot \frac{L_y}{2}}{I_p} \qquad r_{muh} = 4.421 \cdot \frac{kip}{in}$$

$$r_{muv} := \frac{P_u \cdot Ecc \cdot x_{bar}}{I_p}$$ $$r_{muv} = 1.535 \cdot \frac{kip}{in}$$

$$r_{mus} := \frac{P_u}{L_{tot}}$$ $$r_{mus} = 1.333 \cdot \frac{kip}{in}$$

$$r := \sqrt{\left(r_{mus} + r_{muv}\right)^2 + r_{muh}^{\ 2}}$$ $$r = 5.27 \cdot \frac{kip}{in}$$

$$Leg_1 := \frac{r \cdot 1in}{\Phi R_n}$$ $$Leg_1 = 0.237 \cdot in$$

3) Determining the forces on ends A and C

$$r_{muh} := \frac{P_u \cdot Ecc \cdot \frac{L_y}{2}}{I_p}$$ $$r_{muh} = 4.421 \cdot \frac{kip}{in}$$

$$r_{muv} := \frac{P_u \cdot Ecc \cdot \left(L_x - x_{bar}\right)}{I_p}$$ $$r_{muv} = 3.991 \cdot \frac{kip}{in}$$

$$r_{mus} := \frac{P_u}{L_{tot}}$$ $$r_{mus} = 1.333 \cdot \frac{kip}{in}$$

$$r := \sqrt{\left(r_{mus} + r_{muv}\right)^2 + r_{muh}^{\ 2}}$$ $$r = 6.921 \cdot \frac{kip}{in}$$

$$Leg_2 := \frac{r \cdot 1in}{\Phi R_n}$$ $$Leg_2 = 0.311 \cdot in$$

$$Des_{Leg} := \max\left(Leg_1, Leg_2\right)$$ $$Des_{Leg} = 0.311 \cdot in$$

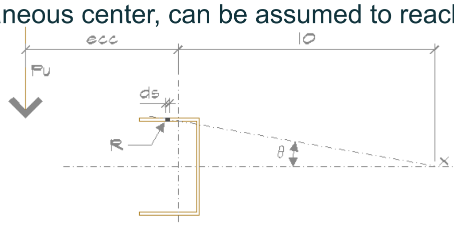
Use 5/16 inch welds at FEXX 70 ksi

Ultimate Strength Method

- A more realistic strength analysis of eccentrically loaded welded connections.
  - The load causes a relative rotation and translation among the parts of the assembly
  - Rotation will take place around the “instantaneous center of rotation” which is dependent upon the geometric forms, the location of the eccentric load, and the deformations that will take place
  - Some parts will yield but some parts will remain and the less stress fibers will contribute more toward the resistance to the load.
  - Failure will occur when all fibers yield

Ultimate Strength Method

- A simple way to visualize this is by considering the least complicated scenario of a weld symmetrically set about the horizontal axis.
  - Each differential element of an infinitesimally small width “ds” in the weld shall provide a resisting force “R.”
  - That force “R” will act perpendicularly to a ray that passes through the instantaneous center.
  - The element that is located the greatest distance from the instantaneous center, can be assumed to reach its limit first.



Ultimate Strength Method

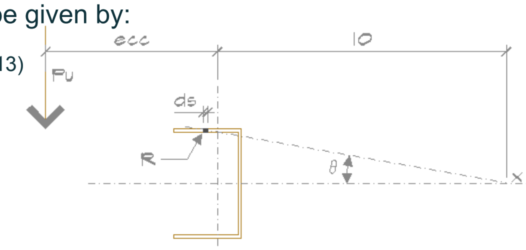
- Deformation:
  - When maximum stress is attained, the deformation Δmax is determined by the following formula:

$$\Delta_{max} = 1.087 w (\theta + 6)^{-0.65} \le 0.17 w \qquad \text{(8-4)}$$

  where w is the weld leg
- Nominal Strength:
  - The nominal strength can be given by:

$$\Phi R_n = 0.75 * C * C_1 * D * l \qquad \text{(8-13)}$$

  - C=tabular value
  - C1=electrode coeff (Table 8-3)
  - D=Leg in sixteenths of inch
  - l=length

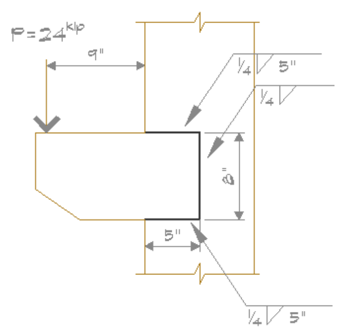


Ultimate Strength Method Example

- Using the same example that was applied for the Elastic method:
  - Reversing the formula for “D”:

$$D = \frac{\Phi R_n}{0.75 * C * C_1 * l}$$

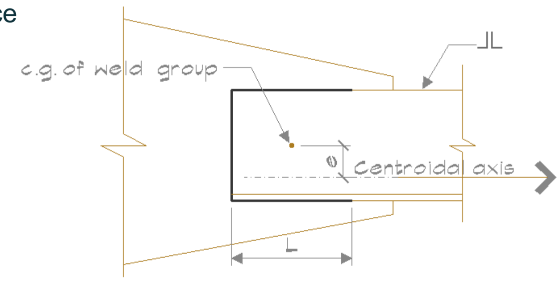
  - Take ΦR_n as $P_u = 24^{kip}$
  - $e_x = 11.611^{in}$
  - $a = e_x / l = 11.61/8 = 1.45$
  - k = 5/8 = 0.625
  - C = 1.42 (by interpolation from table 8-8)
  - C1 = 1 (from table 8-3)



$$D = \frac{\Phi R_n}{0.75 * C * C_1 * l} = \frac{24^{kip}}{0.75 * 1.42 * 1 * 8} = 2.8 \approx 3 \rightarrow 3/16^{inch}$$

Balanced Fillet Welds

- Eccentricity can be generated by designing a weld that has a center of gravity that does not coincide with the centroidal axis of the member:
  - Welds that do not satisfy this criterion are called unbalanced fillet weld connections
  - Eccentricity introduces a moment to the weld group in addition to the axial force

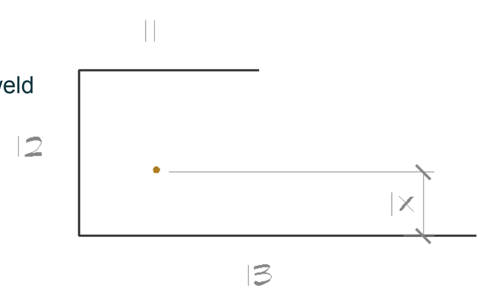


Balanced Fillet Welds

- Calculating the Center of Gravity of a weld group
  - If all the welds are of the same size the factor “w” can be disregarded

$$\bar{x} = \frac{\Sigma\, \bar{x}_i (l_i * w_i)}{\Sigma (l_i * w_i)}$$

  - Where
    - l_i = length of weld
    - w_i = weld leg
    - x_i = distance to c.g. of weld



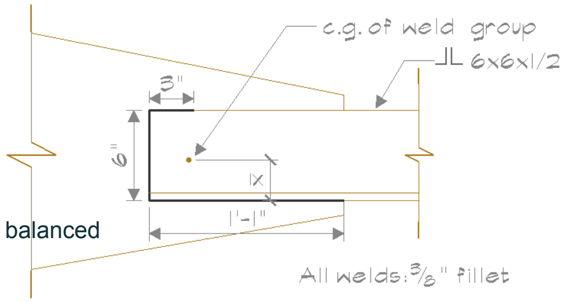
Balanced Fillet Welds In Class Example

- Determine the center of gravity of the weld group and compare to the c.g. of the element to determine if the welds are balanced:
  - The c.g. of the member is at 1.67" (pg. 1-43)
  - Solving:

$$\bar{x} = \frac{\Sigma \bar{x}_i (l_i * w_i)}{\Sigma (l_i * w_i)}$$

$$\bar{x} = \frac{13*0 + 6*3 + 3*6}{13+6+3} = 1.64^{inch}$$

    - Close enough to consider balanced

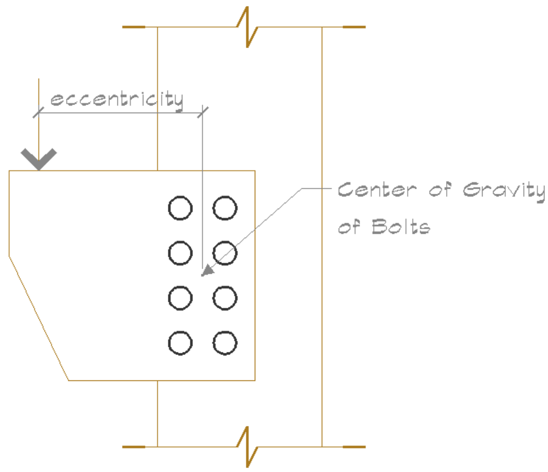


Balanced Fillet Welds

- Per AISC Section J1.7, it is not necessary to balance welds for a statically loaded single angle, double angle, or similar type of element.
- Not using a balanced weld will result in a more compact connection.
- For connections that are subject to cyclic loading a balanced weld design is beneficial.

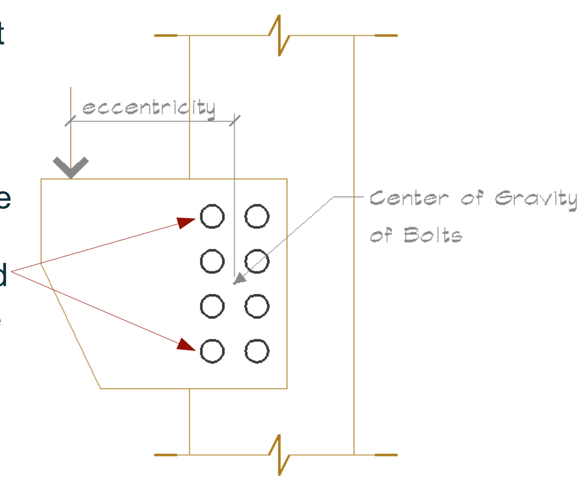
Bolts Subjected to Eccentric Shear

- Bolted eccentrically loaded connections can be analyzed in a manner similar to the methods of analysis of eccentrically loaded welds.
- In certain ways, the analysis is more evident given the clarity of how forces act upon each of the bolts.

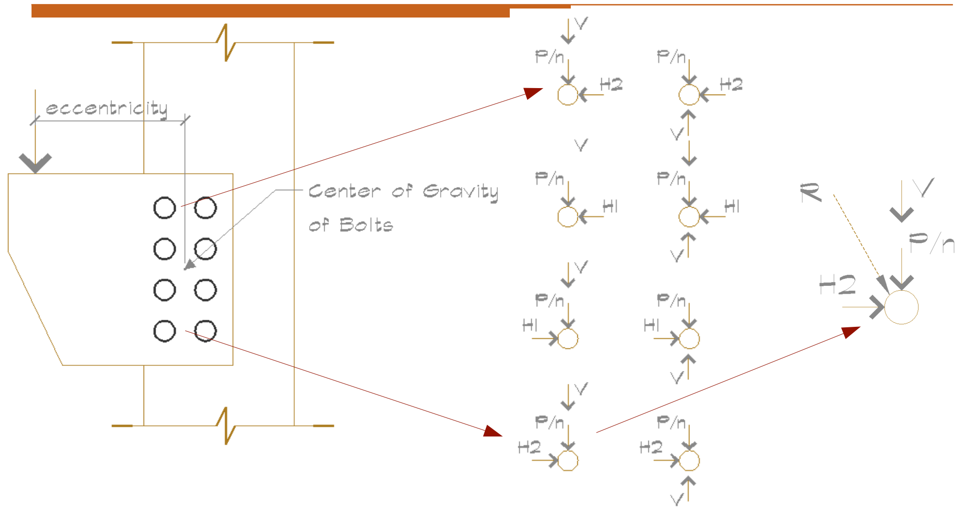


Bolts Subjected to Eccentric Shear

- There are forces that act directly, and forces that are generated from the moment.
- The critical cases are the bolts on the very edge that is closest to the load
  - They are subjected to the direct load, but they also have the longest momentarm and the resultant of the moment adds to the direct load



Eccentric Load on Bolts, Elastic Method

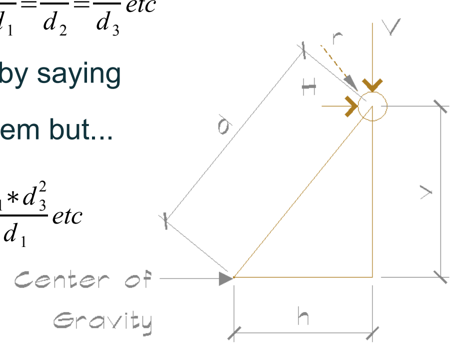


Eccentric Load on Bolts, Elastic Method

- Resultant forces caused on each bolt are proportional to the distance from the cg, ...→ $\frac{r_1}{d_1}=\frac{r_2}{d_2}=\frac{r_3}{d_3} etc$
- That can lead to solve for moment by saying

  $r_1=\frac{r_1*d_1}{d_1}$ redundant as it may seem but...

  this leads to $M=\frac{r_1*d_1^2}{d_1}+\frac{r_1*d_2^2}{d_1}+\frac{r_1*d_3^2}{d_1} etc$



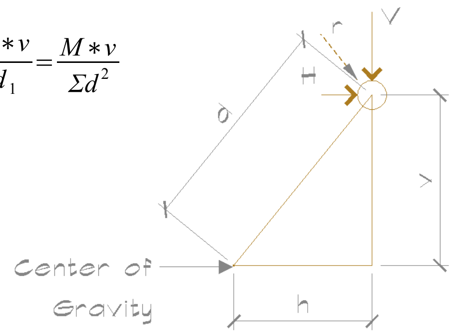
- Thus $M=\frac{r_1*\Sigma d^2}{d_1}$
- and by reversing the equation for the force $r_1=\frac{M*d_1}{\Sigma d^2}$

Eccentric Load on Bolts, Elastic Method

- As we see the Vertical and Horizontal components of force "r" represented by H and V in the diagram →

- $\frac{r_1}{d_1}=\frac{H}{v}$ Which leads to $H=\frac{r_1 * v}{d_1}=\frac{M * v}{\Sigma d^2}$

- And similarly, $V=\frac{r_1 * h}{d_1}=\frac{M * h}{\Sigma d^2}$



Eccentric Load on Bolts, Elastic Method Example

- Determine the force in the most stressed bolt

$$ecc=9.5^{inch}+1.5^{inch}=11^{inch}$$

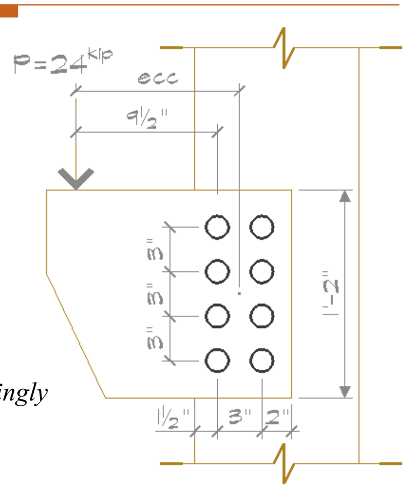
$$M=P_u * ecc=24^{kip} * 11^{inch}=22^{k'}$$

$$\Sigma d^2=\Sigma h^2+\Sigma v^2$$

$$h=1.5^{inch}$$

$$v=1.5^{inch} \; or \; 4.5^{inch} \; accordingly$$

$$\Sigma d^2=\Sigma h^2+\Sigma v^2$$



Eccentric Load on Bolts, Elastic Method Example

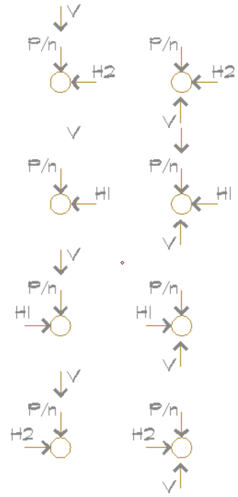
- $\Sigma d^2 = 8*(1.5)^2 + 4*(1.5^2 + 4.5^2) = 108$
- Taking lower left bolt...

$$H = \frac{M*v}{\Sigma d^2} = \frac{264^{k^{inch}} * 4.5^{inch}}{108^{inch}} = 11^{kip} \leftarrow$$

$$V = \frac{M*h}{\Sigma d^2} = \frac{264^{k^{inch}} * 1.5^{inch}}{108^{inch}} = 3.67^{kip} \downarrow$$

$$\frac{P_u}{n} = \frac{24^{kip}}{8} = 3^{kip} \downarrow$$

$$R = \sqrt{11^2 + (3.67+3)^2} = 12.86^{kip} \qquad \Phi R_n = 17.146^{kip}$$



Instantaneous Center of Rotation Method

- Just like seen in welds, the elastic method is based on the assumption that the fasteners are behaving elastically and that there are no other factors besides the stiffness of the connectors.
- The Instantaneous center of rotation method gives less conservative and more realistic result.
- Using Table 7-7 (for 3") ...the process is much quicker:
  - $e_x = 11^{in}$
  - $s = 3^{in}$
  - n=4
  - By interpolation C=2.24

$$\Phi R_n = \frac{P_u}{C} = \frac{24^{kip}}{2.24} = 10.71^{kip}$$

- Using Table 7-1 a bolt type can be selected.
  - Take Group A ⅝" diameter!