Mohr's Circle
Method of Analysis for Principal Stresses

A lecture assembled for the course on
Statics and Strength of Materials

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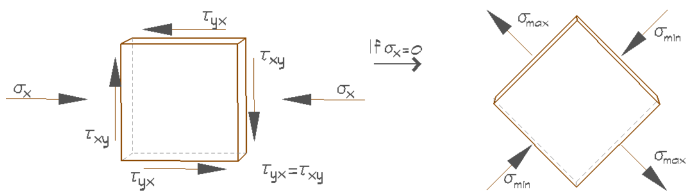
Intro

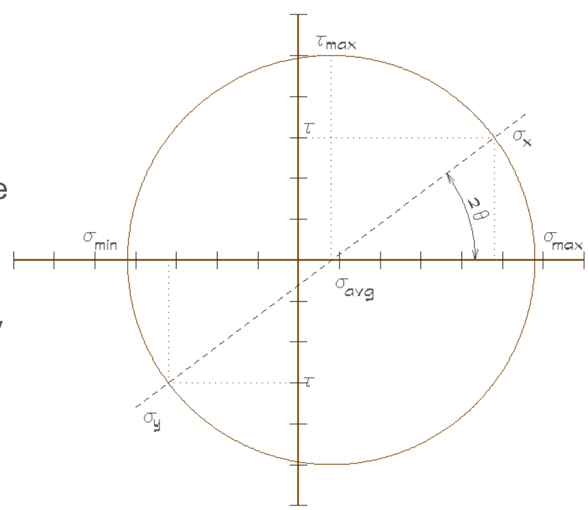
MOHR'S CIRCLE

- A method of analysis for principal stresses
  - It has become evident that it is possible to determine a direction at which the surfaces of an infinitesimally small element within a body can be subjected to pure normal stresses and no shear.
  - Naturally, the scenario of pure shear and no normal stresses whatsoever may be less than likely. It is necessary to understand this behavior of materials in order to perceive how and why structural elements are designed the way they are, and how reinforcement may be applied in order to address the weaknesses of the materials.
  - The design portion of this topic will be addressed in a later. At this point it is necessary to focus on the analysis and the structural behavior in order to determine where a structure will be subjected to pure Tension and Compression that will facilitate the designer to make the appropriate decisions for sizing and reinforcing a structural element such as a beam, a column, etc.



MOHR'S CIRCLE

- A method of analysis for principal stresses
  - A very efficient graphical method to determine the direction of the principal stresses and the maximum shear that an element is subjected to is that of Mohr's circle.
  - The relations of the normal and the shear stresses are perfectly set within the geometry of a circle the center of which is located along the abscissa which represents the values of the normal stresses, whilst the ordinates represent the shear stress values at any given angle.
  - With the center of the circle on the abscissa it is granted that the principal stresses (σ_{max} and σ_{min}) will be on the opposite points along the diameter.
  - The maximum shear stress represented on the ordinates will yield the maximum value equal to the radius. Thus the center of the circle is given by:



$$\sigma_{avg}=\frac{(\sigma_x+\sigma_y)}{2}$$

Mohr's Circle

- A method of analysis for principal stresses
  - Once the center is determined, it is necessary to evaluate the Radius of the circle which can be given by:

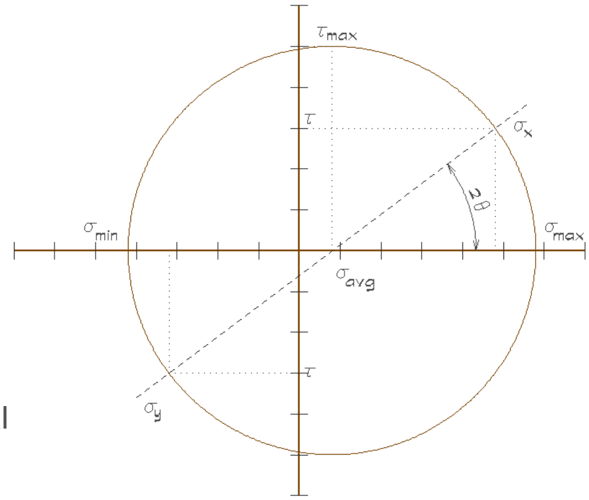
$$R=\sqrt{\left[\frac{\sigma_x-\sigma_y}{2}\right]^2+\tau_{xy}^2}$$

  - Therefore, the values of maximum and minimum normal stresses can be given by:

$$\sigma_{max}, \sigma_{min}=\sigma_{avg}\pm R$$

  - As previously stated, the maximum shear will be equal to the radius of the circle, thus:

$$\tau_{max}, \tau_{min}=\pm R$$

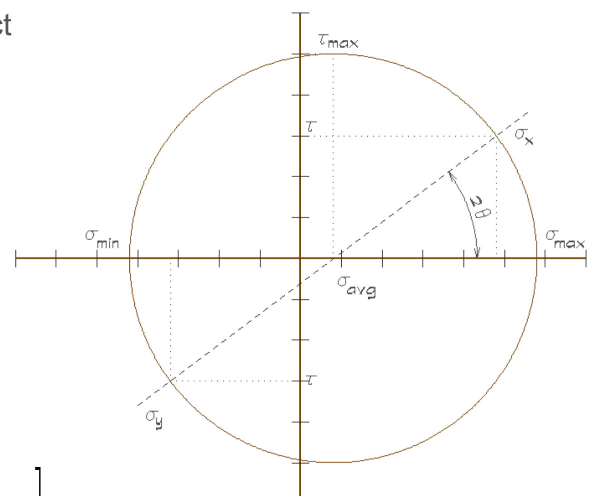


Mohr's Circle

- A method of analysis for principal stresses
  - A part that may cause some initial confusion is the fact that whilst in reality the directions of σ_{max} and σ_{min} or even σ_x and σ_y are perpendicular, on the diagram of Mohr's circle they are on opposite direction along the same axis, i.e. the 90º in reality are represented as 180º on Mohr's circle.
  - Therefore, it would be logical to anticipate that the angle of rotation at which the principal stresses occur would be also double on the diagram of Mohr's circle.
  - The relation that gives the value or angle θ is the following:

$$\tan 2\theta=\frac{2\tau_{xy}}{\sigma_x-\sigma_y}$$

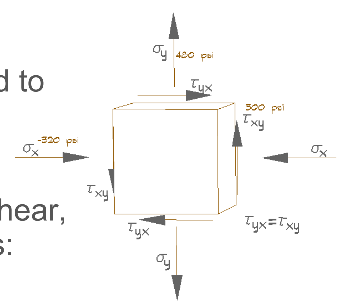
  - Which could also be expressed as: $\theta=0.5\cdot atan\left[\frac{2\tau_{xy}}{\sigma_x-\sigma_y}\right]$

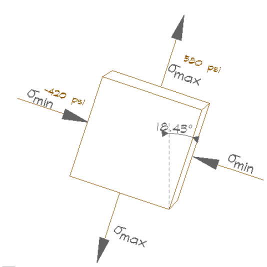


In Class Example

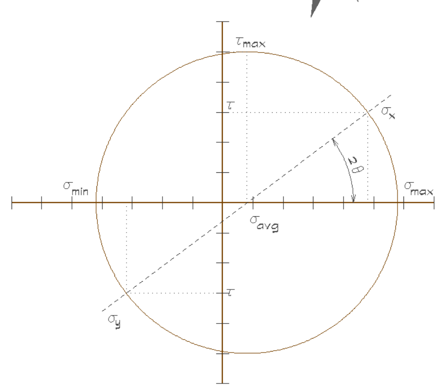
EXAMPLE ON MOHR'S CIRCLE

- A method of analysis for principal stresses
  - An infinitesimally small cubic element is subjected to the following stresses:
    - τ_{xy} = 300^{psi} σ_x = -320^{psi} σ_y = 480^{psi}
  - Determine the principal stresses, the maximum shear, and the angle of rotation for the principal stresses:





  - Initially the average stress or center of Mohr's circle can be determined, and then the radius of the circle:



$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-320^{psi} + 480^{psi}}{2} = 80^{psi}$$

$$R = \sqrt{\left[\frac{\sigma_x - \sigma_y}{2}\right]^2 + \tau_{xy}^2} = \sqrt{\left[\frac{-320^{psi} - 480^{psi}}{2}\right]^2 + (300^{psi})^2} = 500^{psi} = \tau_{max}$$

- An example on principal stress analysis
  - Solving for the principal stresses:

$$\sigma_{max} = \sigma_{avg} + R = 80^{psi} + 500^{psi} = 580^{psi}$$

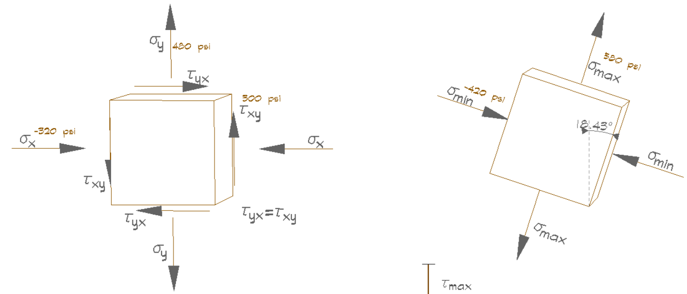
&

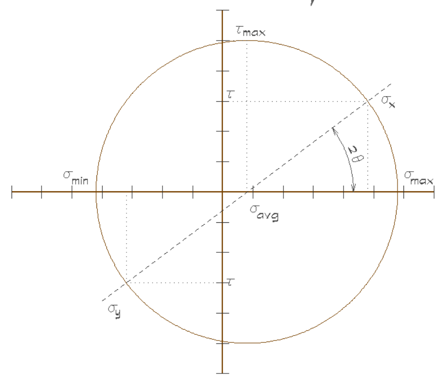
$$\sigma_{min} = \sigma_{avg} - R = 80^{psi} - 500^{psi} = -420^{psi}$$

  - Finally, to determine the angle of rotation for the principal stresses it may be convenient to use the second of the two available formulae:

$$\theta = 0.5 \cdot atan\left[\frac{2 \cdot \tau_{xy}}{\sigma_x - \sigma_y}\right] = 0.5 \cdot atan\left[\frac{2 \cdot 300^{psi}}{-320^{psi} - 480^{psi}}\right] = -18.43^{o}$$

  - For purposes of convenience, the diagram of Mohr's circle in the previous page perfectly corresponds to the values of this exercise.

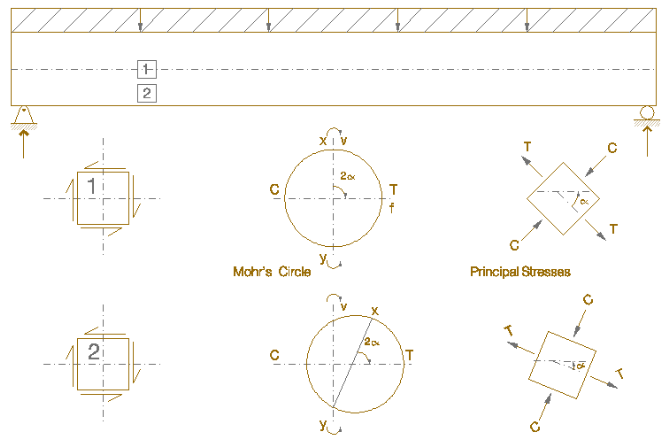




In Conclusion

How is it Applicable

- Susceptibility of flexural elements to shear stress:
  - Given the above mentioned data, let's consider two infinitesimally small elements "1" and "2" along a simply supported beam and take the stresses developed.
  - Granted the symmetrical form of the beam, two points are adequate to generate a pattern. Between point "1" and point "2" we see a shift in the direction of the principal stresses.
  - It can be deduced that the direction along which the principal stresses occur follows a path.
  - Furthermore, it can be suggested that the cracks to the material will happen at a direction that is perpendicular to the direction of the tension.



How is it Applicable

- Susceptibility of flexural elements to shear stress:
  - Finally if we consider that the maximum shear stress occurs at the ends (support points) of a simply supported beam that is subjected to a uniformly distributed load, and exactly at midpoint is subjected to no shear stress, the following diagram can be generated. A set of tensile stress trajectories is formed based on the results given by the above example. ...